Some Quantum Aspects of Complex Vector Fields with Chern-Simons Term

O.M. Del Cima*†
Brazilian Centre for Physical Researches (CBPF)
Department of Fields and Particles (DCP)
Rio de Janeiro - RJ - Brazil.

F.A.B. Rabelo de Carvalho
Catholic University of Petrópolis (UCP) - ICEN
Petrópolis - RJ - Brazil.

Abstract

Complex vector fields with Maxwell, Chern-Simons and Proca terms are minimally coupled to an Abelian gauge field. The consistency of the spectrum is analysed and 1-loop quantum corrections to the self-energy are explicitly computed and discussed. The incorporation of 2-loop contributions and the behaviour of tree-level scattering amplitudes in the limit of high center-of-mass energies are also commented.

*Bitnet e-mail: oswaldo@brlncc.bitnet
†Internet e-mai: oswaldo@cbpfsu1.cat.cbpf.br

1 Introduction

One of the central problems in the framework of gauge field theories is the issue of gauge field mass. Gauge symmetry is not, in principle, conflicting with the presence of a massive gauge boson. In 2 space-time dimensions, the well-known Schwinger model puts in evidence the presence of a massive photon without the breaking of gauge symmetry [1]: a dynamical mass generation takes place by virtue of fermion 1-loop corrections to the Maxwell field polarization tensor.

Another evidence for the compatibility between gauge symmetry and massive vector fields comes from the study of 3-dimensional gauge theories [2, 3]. A topological mass term referred to as the Chern-Simons Lagrangian, once added to the Maxwell kinetic term, shifts the photon mass to a non-vanishing value without breaking gauge invariance at all [2, 3]. Even if the Chern-Simons term, which is gauge invariant, is not written down at tree-level, it may be generated by 1-loop corrections whenever massive fermions are minimally coupled to an Abelian gauge field [4, 5, 6]. Again, a dynamical mass generation mechanism takes place. Also, in 3 space-time dimensions, there occurs a dynamical fermionic mass generation if massless fermions are minimally coupled to a Chern-Simons field [4, 5, 6, 7].

In the more realistic case of 4 space-time dimensions, the best mechanism known, up to now, to solve the problem of intermediate boson masses is the spontaneous symmetry breaking mechanism [8, 9]. It is not known any 4-dimensional counterpart of the dynamical mechanism to generate gauge field masses along the lines previously mentioned. However, in 4 dimensions, one should quote the dynamical breaking of chiral symmetry which takes place through a dynamical mass generation mechanism for fermions [10, 11].

Since, over the past years, 3-dimensional field theories have been shown to play a central rôle in connection with the behaviour of 4-dimensional theories at finite temperature [12] and in the description of a number of problems in Condensed Matter Physics [13], it seems reasonable to concentrate efforts in trying to understand some peculiar features of gauge field dynamics in 3 dimensions. Also, the recent result on the Landau gauge finiteness of Chern-Simons theories is a remarkable property that makes 3-dimensional gauge theories so attractive [14].

The main purpose of this paper is to consider 3-dimensional models built up in terms of complex vector fields with Chern-Simons terms and to which

one minimally couples a Maxwell field. At tree-level, we study the Chern-Simons-Maxwell (CSM*) and the Chern-Simons-Maxwell-Proca (CSMP*) cases, in order to analyse the conditions to be set on the free parameters of the Lagrangians, so as to avoid the presence of tachyons and ghosts in the spectrum. This is carried out in Section 2. In Section 3, we study the Abelian CSM* model and show that, upon the incorporation of 1-loop corrections to the CSM*-field self-energy, a finite Proca mass term is generated. The analysis of Section 2, in combination with the latter result, ensures that the generated Proca-like term does not plug the theory with tachyons or ghosts. Finally, in Section 4, we discuss the incorporation of 2-loops contributions to the model, some results concerning the behaviour of the scattering amplitudes in the limit of very high center-of-mass energies are discussed and we draw our general conclusions. One Appendix follows where the explicit results for the momentum-space 1-loop integrals are collected. The metric adopted throughout this work is $\eta_{\mu\nu} = (+; -, -)$.

2 The Complex Chern-Simons-Maxwell (CSM*) and Chern-Simons-Maxwell-Proca (CSMP*) Fields

The CSM* model is described by the Lagrangian

$$\mathcal{L}^0_{CSM} = \frac{1}{2}\epsilon^{\alpha\mu\nu} B^*_\alpha G_{\mu\nu} - \frac{1}{2M} G^*_{\mu\nu} G^{\mu\nu} \quad , \tag{1}$$

where $G_{\mu\nu}\equiv\partial_\mu B_\nu - \partial_\nu B_\mu$ and $G^*_{\mu\nu}\equiv\partial_\mu B^*_\nu - \partial_\nu B^*_\mu$ are the field-strengths, and M is a real parameter with dimension of mass.

There are two kinds of $U(1)$ symmetries that may be observed in (1). A global $U_\alpha(1)$ given by

$$B'_\mu(x) = e^{i\alpha} B_\mu(x) \quad , \tag{2}$$

where α is a real parameter, and a local $U_\beta(1)$ that reads

$$B'_\mu(x) = B_\mu(x) + \partial_\mu \beta(x) \quad , \tag{3}$$

where $\beta(x)$ is an arbitrary C^∞ complex function. The question involving gauge symmetries with complex parameters has already been contemplated

in the context of spontaneously broken symmetries in supersymmetric gauge models [15].

To minimally couple the CSM* fields, B_μ and B^*_μ, to the Maxwell field, A_μ, we define the following $U_\alpha(1)$-covariant derivatives :

$$D_\mu \equiv \partial_\mu + i\omega A_\mu \ \text{ and } \ D^*_\mu \equiv \partial_\mu - i\omega A_\mu \ , \tag{4}$$

where ω is a coupling constant with dimension of $(\text{mass})^{\frac{1}{2}}$. Then, the total Lagrangian becomes

$$\begin{aligned} \mathcal{L}_{CSM}(B, B^*, \partial B, \partial B^*, A) &= \mathcal{L}^0_{CSM}(B, B^*, DB, D^*B^*) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &= \frac{1}{2}\epsilon^{\alpha\mu\nu}B^*_\alpha \tilde{G}_{\mu\nu} - \frac{1}{2M}\tilde{G}^*_{\mu\nu}\tilde{G}^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \ , \end{aligned} \tag{5}$$

where $\tilde{G}_{\mu\nu}{\equiv}D_\mu B_\nu{-}D_\nu B_\mu$, and $F_{\mu\nu}$ is the field-strength for A_μ. By replacing the covariant derivatives as given in eq.(4), the total Lagrangian reads :

$$\begin{aligned} \mathcal{L}_{CSM} \ = \ & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\epsilon^{\alpha\mu\nu}B^*_\alpha G_{\mu\nu} - \frac{1}{2M}G^*_{\mu\nu}G^{\mu\nu} + i\omega\epsilon^{\alpha\mu\nu}B^*_\alpha A_\mu B_\nu + \\ & -i\frac{\omega}{M}(G^*_{\mu\nu}A^\mu B^\nu - G_{\mu\nu}A^\mu B^{*\nu}) - \frac{\omega^2}{M}(A_\mu B_\nu - A_\nu B_\mu)A^\mu B^{*\nu} \ . \end{aligned} \tag{6}$$

It can be noticed that the local $U_\beta(1)$-symmetry (3) is explicitly broken by the interaction terms in (6).

In order to perform the analysis of the spectral consistency of this model, it is necessary to obtain the propagator for the fields B and B^*. Since the local $U_\beta(1)$-symmetry is broken only at the interaction level, we need a gauge-fixing term to be able to read off the propagators. So, for the sake of extracting them, we consider the Lagrangian below :

$$\hat{\mathcal{L}}^0_{CSM} = \frac{1}{2}\epsilon^{\alpha\mu\nu}B^*_\alpha G_{\mu\nu} - \frac{1}{2M}G^*_{\mu\nu}G^{\mu\nu} + \frac{1}{\hat{\alpha}}(\partial_\mu B^{*\mu})(\partial_\nu B^\nu) \ , \tag{7}$$

where $\hat{\alpha}$ is the gauge-fixing parameter.

The field equations coming from (7) are given by

$$\mathcal{O}^{\epsilon\alpha}B^*_\alpha = 0 \ , \tag{8}$$

with

$$\mathcal{O}^{\epsilon\alpha} \equiv -\epsilon^{\epsilon k\alpha}\partial_k - \frac{\Box}{M}\left(\eta^{\epsilon\alpha} - \frac{\partial^\epsilon\partial^\alpha}{\Box}\right) + \frac{\Box}{\hat{\alpha}}\left(\frac{\partial^\epsilon\partial^\alpha}{\Box}\right) \ , \tag{9}$$

	Ω	Θ	S
Ω	Ω	$\mathbf{0}$	$\mathbf{0}$
Θ	$\mathbf{0}$	Θ	S
S	$\mathbf{0}$	S	$-\Box\,\Theta$

Table 1: Operator algebra fulfilled by Ω,Θ and S.

where

$$\Theta^{\mu\nu} \equiv \eta^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{\Box}\ , \quad S^{\mu\nu} \equiv \epsilon^{\mu\alpha\nu}\partial_\alpha \ \text{ and } \ \Omega^{\mu\nu} \equiv \frac{\partial^\mu \partial^\nu}{\Box} \tag{10}$$

are spin operators that fulfil the algebra displayed in Table 1.

Inverting the operator $\mathcal{O}$ with the help of the Table 1, we obtain the following momentum-space propagators in the longitudinal and transverse subspaces, respectively :

$$\Delta_L^{\mu\nu}(k) \ = \ i\,\frac{\hat{\alpha}}{k^2}\left(\frac{k^\mu k^\nu}{k^2}\right) \tag{11.a}$$

and

$$\Delta_T^{\mu\nu}(k) \ = \ -i\,\frac{M^2}{k^2(k^2-M^2)}\left[i\,\epsilon^{\mu k\nu}k_k + \frac{k^2}{M}\left(\eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\right] . \tag{11.b}$$

By saturating the propagators with external conserved currents, J^μ and $J^{\mu *}$, the following result on the spectrum can be stated :

$$L-sector \longrightarrow \text{pole at } k^2 = 0 \text{ non-dynamical} \tag{12.a}$$

$$T-sector \longrightarrow \begin{cases} \text{pole at } k^2 = 0 \text{ non-dynamical} \\ \text{pole at } k^2 = M^2 \begin{cases} \text{dynamical} \\ \text{no tachyons, no ghosts } \ if \ \ M \ > \ 0 \end{cases} \end{cases} . \tag{12.b}$$

Thus, we may conclude that, once the mass parameter, M, is taken to be positive, the CSM* model describes a free physical dynamical excitation of mass $k^2 = M^2$.

The CSMP* model is described by a Lagrangian obtained from (1) by the addition of a Proca term, $\hat{\mu}B^*_\mu B^\mu$. Then,

$$\mathcal{L}^o_{CSMP} = \frac{1}{2}\epsilon^{\alpha\mu\nu}B^*_\alpha G_{\mu\nu} - \frac{1}{2M}G^*_{\mu\nu}G^{\mu\nu} + \hat{\mu}B^*_\mu B^\mu \ , \tag{13}$$

where $\hat{\mu}$ is a real parameter with mass dimension.

It may be observed that the Lagrangian of eq.(13) exhibits only one global symmetry, $U_\alpha(1)$:

$$B'_\mu(x) = e^{i\alpha}B_\mu(x) \ \ , \tag{14}$$

where α is a real parameter. The local symmetry $U_\beta(1)$ (3) is explicitly broken by the Proca term.

Carrying out the minimal coupling of the CSMP* fields, B_μ and B^*_μ, to the Maxwell field A_μ, one gets the Lagrangian

$$\begin{aligned}\mathcal{L}_{CSMP} \ &= \ -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\epsilon^{\alpha\mu\nu}B^*_\alpha G_{\mu\nu} - \frac{1}{2M}G^*_{\mu\nu}G^{\mu\nu} + \hat{\mu}B^*_\mu B^\mu + \\ &\quad + i\omega\epsilon^{\alpha\mu\nu}B^*_\alpha A_\mu B_\nu - i\frac{\omega}{M}(G^*_{\mu\nu}A^\mu B^\nu - G_{\mu\nu}A^\mu B^{*\nu}) + \\ &\quad -\frac{\omega^2}{M}(A_\mu B_\nu - A_\nu B_\mu)A^\mu B^{*\nu} \ .\end{aligned} \tag{15}$$

To pursue our investigation on the consistency of the spectrum, we shall now quote the expressions derived for the propagators of the CSMP* fields and then analyse their poles and associated residues.

The momentum-space expressions for the propagators are :

$$\overline{\Delta}^{\mu\nu}_L(k) \ = \ i\,\frac{1}{\hat{\mu}}\left(\frac{k^\mu k^\nu}{k^2}\right) \tag{16.a}$$

and

$$\begin{aligned}\overline{\Delta}^{\mu\nu}_T(k) \ &= \ -i\,\frac{M}{[(k^2-\hat{\mu}M)^2 - M^2k^2]}\left[iM\epsilon^{\mu\kappa\nu}k_\kappa + (k^2-\hat{\mu}M)\left(\eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\right] \\ &= \ -i\,\frac{M}{(k^2-m^2_+)(k^2-m^2_-)}\left[iM\epsilon^{\mu\kappa\nu}k_\kappa + (k^2-\hat{\mu}M)\left(\eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\right] \ ,\end{aligned} \tag{16.b}$$

where

$$m_+^2 \;\equiv\; \frac{M}{2}\,[M+2\hat{\mu}+\sqrt{M(M+4\hat{\mu})}\,] \tag{17.a}$$

and

$$m_-^2 \;\equiv\; \frac{M}{2}\,[M+2\hat{\mu}-\sqrt{M(M+4\hat{\mu})}\,]\;, \tag{17.b}$$

with $M(M+4\hat{\mu}) \geq 0$, in order to avoid unphysical complex roots.

To avoid the apperance of a double pole, $m_+^2 = m_-^2$ (which would certainly lead to a ghost), we must actually have $M(M+4\hat{\mu}) > 0$.

Again, we saturate the propagators with external conserved currents, and the following results on the spectrum hold :

$$T\!-\!sector \longrightarrow \begin{cases} \text{pole at } k^2 = m_+^2 \begin{cases} \text{dynamical} \\ \text{no tachyons, no ghosts } \; if \;\; M \text{ and } \hat{\mu} > 0 \end{cases} \\ \text{pole at } k^2 = m_-^2 \begin{cases} \text{dynamical} \\ \text{no tachyons, no ghosts } \; if \;\; M \text{ and } \hat{\mu} > 0 \end{cases} \end{cases} . \tag{18}$$

The analysis of the residues shows that the T-sector is free from tachyons and ghosts whenever $\hat{\mu} > 0$ and $M > 0$.

Also, the conditions $\hat{\mu} > 0$ and $M > 0$ automatically avoid a double pole. Then, the CSMP* model is perfectly physical, as long as the spectrum is concerned, if these two conditions are set.

Nevertheless, to control the issue of unitarity at tree-level, it is necessary to study the behaviour of scattering cross sections in the limit of very high center-of-mass energies. This has been discussed in detail in the paper of ref.[16], where we have illustrated that the scattering process between a CSM* vectorial boson and the photon exhibits a cross-section whose asymptotic behaviour respects the Froissart bound in 3 dimensions.

A peculiar feature concerns the presence of two different simple poles in the transverse sector of the propagator for the CSMP*-field. This is also a characteristic of a real CSMP-field. The poles are to be interpreted as two distinct excitations whose spins have to be fixed in terms of the masses, after a detailed analysis of the Lorentz group generators as functionals of the fields is carried out, in the same way it is done for a topologically massive theory [3]. However, each of the masses has a definite value for the spin, ± 1, since there is no room for different polarization states in $D = 3$. Hence,

the 2 degrees of freedom of the real CSMP-field correspond to the 2 possible values for the mass, $m^2_\pm$. In the complex case, the 4 degrees of freedom are associated to the 2 different states of charge that each massive pole may present.

3 Dynamical Mass Generation in the CSM* Model

By reconsidering the Lagrangian (6), the following interaction vertices (see Fig.1) come out :

$$\begin{aligned}\mathcal{L}^{(1)\,int}_{CSM} &= i\,\omega\epsilon^{\alpha\mu\nu}\,B^*_\alpha A_\mu B_\nu \quad\longrightarrow\quad V_3\;, &(19.a)\\ \mathcal{L}^{(2)\,int}_{CSM} &= -i\,\frac{\omega}{M}\,(G^*_{\mu\nu}A^\mu B^\nu - G_{\mu\nu}A^\mu B^{*\nu}) \quad\longrightarrow\quad \overline{V}_3 &(19.b)\end{aligned}$$

and

$$\mathcal{L}^{(3)\,int}_{CSM} = -\,\frac{\omega^2}{M}\,(A_\mu B_\nu - A_\nu B_\mu)A^\mu B^{*\nu} \quad\longrightarrow\quad V_4\;. \qquad (19.c)$$

Before the calculation of the Feynman graphs relevant for our analysis on the mass generation, we present the expression we get for the superficial degree of divergence of the primitively divergent graphs of the model.

Analysing the CSM* propagator in the high energy limit, and taking into account the interaction vertices above, we find the following expression for the superficial degree of divergence, δ_{CSM} :

$$\delta_{CSM} = 3 - \frac{3}{2}v_3 - \frac{1}{2}\overline{v}_3 - v_4 - \frac{1}{2}E_A - \frac{1}{2}E_B\;, \qquad (20)$$

where v_3, $\overline{v}_3$ and v_4 are the numbers of vertices V_3, $\overline{V}_3$ and V_4 respectively, E_A are the external lines of A_μ and E_B are the external lines of B_μ and B^*_μ. Therefore, the CSM* is a super-renormalizable model: ultraviolet divergences appear only up to 2-loops. Now, since in 3 space-time dimensions no 1-loop divergences show up, all renormalizations have to be performed at 2-loops.

The vertex Feynman rules of the model read as below :

$$(V_3)_{\alpha\mu\nu} = \omega\epsilon_{\alpha\mu\nu}\;, \qquad (21.a)$$

$$(\overline{V}_3)_{\alpha\mu\nu} = i\frac{\omega}{M}(\eta_{\nu\alpha}k_\mu - \eta_{\mu\alpha}k_\nu + \eta_{\nu\alpha}m_\mu - \eta_{\mu\nu}m_\alpha) \tag{21.b}$$

and

$$(V_4)_{\alpha\nu\beta\mu} = i\frac{2\omega^2}{M}(\eta_{\alpha\beta}\eta_{\mu\nu} - \eta_{\alpha\nu}\eta_{\beta\mu}) \,. \tag{21.c}$$

In Fig.2, we list the 1-loop diagrams that contribute to the CSM*-field self-energy. The explicit results for these diagrams may be found in ref.[17], where computations have been carried out in Landau gauge, $\hat{\alpha}$=0.

Bearing in mind that we are concerned with the possibility of inducing a 1-loop (finite) mass contribution of the Proca type, we can select only those terms that do not exhibit any dependence on the external momenta and are moreover symmetric on the free indices of the external lines. Therefore, the only terms that potentially contribute a finite Proca mass term have been found to be given by the following parametric integrals :

$$\begin{aligned}
(I_1)_{\alpha\beta} &= \frac{\omega^2}{M}\int\frac{d^3k}{(2\pi)^3}\frac{k_\alpha k_\beta}{(k-p)^2(k^2-M^2)} \\
&= i\frac{\omega^2}{8\pi M}\Big\{p_\alpha p_\beta\int_0^1 dx\frac{x^2}{[p^2x^2-(p^2+M^2)x+M^2]^{\frac{1}{2}}}+ \\
&\quad + \eta_{\alpha\beta}\int_0^1 dx[p^2x^2-(p^2+M^2)x+M^2]^{\frac{1}{2}}\Big\}\,, && (22.a) \\
(I_2)_{\alpha\beta} &= \frac{\omega^2}{M}\eta_{\alpha\beta}\int\frac{d^3k}{(2\pi)^3}\frac{k^2}{(k-p)^2(k^2-M^2)} \\
&= i\frac{\omega^2}{8\pi M}\eta_{\alpha\beta}\Big\{p^2\int_0^1 dx\frac{x^2}{[p^2x^2-(p^2+M^2)x+M^2]^{\frac{1}{2}}}+ \\
&\quad + 3\int_0^1 dx[p^2x^2-(p^2+M^2)x+M^2]^{\frac{1}{2}}\Big\}\,, && (22.b) \\
(I_3)_{\alpha\beta} &= M\omega^2\eta_{\alpha\beta}\int\frac{d^3k}{(2\pi)^3}\frac{1}{(k-p)^2(k^2-M^2)} \\
&= i\frac{\omega^2 M}{8\pi}\eta_{\alpha\beta}\int_0^1 dx\frac{1}{[p^2x^2-(p^2+M^2)x+M^2]^{\frac{1}{2}}} && (22.c)
\end{aligned}$$

and

$$
\begin{aligned}
(I_4)_{\alpha\beta} &= M\omega^2 \int \frac{d^3k}{(2\pi)^3} \frac{k_\alpha k_\beta}{(k-p)^2(k^2-M^2)k^2} \\
&= i\frac{\omega^2}{8\pi M}\Big\{ p_\alpha p_\beta \int_0^1 dx \frac{x^2}{[p^2x^2-(p^2+M^2)x+M^2]^{\frac{1}{2}}} + \\
&\quad + \eta_{\alpha\beta}\int_0^1 dx [p^2x^2-(p^2+M^2)x+M^2]^{\frac{1}{2}} + \\
&\quad - p_\alpha p_\beta \int_0^1 dx \frac{x^2}{(p^2x^2-p^2x)^{\frac{1}{2}}} - \eta_{\alpha\beta}\int_0^1 dx (p^2x^2-p^2x)^{\frac{1}{2}} \Big\} \qquad \text{(22.d)}
\end{aligned}
$$

Their explicit results are presented in the Appendix. By observing these results (see 30, 31, 32 and 33), we conclude that a 1-loop term given by $i\frac{\omega^2}{32\pi}\eta_{\alpha\beta}$, coming from I_1 and I_4, will lead to the generation of the Proca term.

The whole 1-loop CSM* self-energy diagram, $\Omega^{(1)}$, is the sum of the diagrams Σ, Λ, Ξ^R, Ξ^L and Γ of Fig.2 :

$$
\Omega^{(1)} = \Sigma + \Lambda + \Xi^R + \Xi^L + \Gamma \ . \tag{23}
$$

By summing up all these pieces, we finally get that the 1-loop induced Proca term comes from the contribution

$$
\Omega^{(1)\alpha\beta}_{\hat{\mu}} = i\,\frac{\omega^2}{8\pi}\,\eta^{\alpha\beta} = i\,\hat{\mu}\,\eta^{\alpha\beta} \ , \tag{24}
$$

from which we can readily read the Proca mass :

$$
\hat{\mu} = \frac{\omega^2}{8\pi} > 0 \ . \tag{25}
$$

It is interesting to emphasize that the term $\Omega^{(1)}_{\hat{\mu}}$, generated by the 1-loop quantum corrections to the CSM* self-energy, is a finite one, therefore it will not be necessary to add any counter-term to the Lagrangian (6). Such a finite term amounts to the contribution

$$
\mathcal{L}^{(1)}_{\hat{\mu}} = \hat{\mu} B^*_\mu B^\mu \tag{26}
$$

to the classical Lagrangian. Since the parameter $\hat{\mu}$ in $\mathcal{L}^{(1)}_{\hat{\mu}}$ automatically satisfies the condition $\hat{\mu} > 0$, the espectral consistency discussed in the previous section is not jeopardized.

Figure 1: Interaction 3- and 4-vertices, V_3, $\overline{V}_3$ and V_4.

Figure 2: 1-loop CSM*-field self-energy diagrams.

4 Discussions and General Conclusions

Our basic proposal in this paper has been to understand a number of features concerning the dynamics of complex vector fields in D=1+2.

The first step of our study consisted in establishing conditions under which a general CSMP complex vector field describes physically acceptable excitations. It was obtained that such a complex vector field describes, in principle, two distinct massive excitations, each of them appearing of course in two states with opposite charges.

Having understood how to control the physical character of the quanta of the model, we proposed to study the dynamics of a CSM*-field minimally coupled to an Abelian vector field (Maxwell field). The explicit calculation of 1-loop corrections revealed the generation of a (finite) Proca term that was not present at tree-level, respecting the spectral conditions set on the study of the propagation of the CSMP*-field. We then concluded that the 1-loop Proca mass generation does not introduce neither tachyons nor ghosts in the spectrum.

The study concerning the behaviour of the "Compton" scattering cross sections in the limit of very high (much higher than the masses of the quanta) center-of-mass energies revealed that the CSM*-model respects the Froissart bound for D=1+2, while the CSMP*-model violates this bound. Moreover, contrary to what happens in the case of 4-dimensional massive charged vector fields coupled to the Maxwell-field, Froissart bound cannot be restored at the expenses of a gauge-invariant non-minimal coupling [16].

Also, another delicate point should be discussed. The CSM*-model presents divergences at the 2-loop level. Therefore, it is crucial to check whether or not a ultraviolet divergent term of the form $|(\partial_\mu B^\mu)|^2$ appears as a 2-loop contribution to the CSM*-field self-energy. In view of this result, one may have to add, for the sake of renormalization, the term $|(\partial_\mu B^\mu)|^2$ already at the classical level, and ghosts will unavoidably show up that spoil the spectrum [18]. To clarify this matter, one has to investigate the 2-loop self-energy graphs for the CSM*-field displayed in Fig.3.

Nevertheless, based on the power-counting (20) derived for the model we are considering, we find out that the graphs drawn in Fig.3 that involve exclusively $\overline{V}_3$ and V_4 vertices are all logarithmically divergent (δ_{CSM}=0). On the other hand, since a $p^\mu p^\nu$-dependence has to be factored out from these graphs so as to build up a 2-loop correction of the form $|(\partial_\mu B^\mu)|^2$, this sort

Figure 3: 2-loop CSM*-field self-energy logarithmically divergent diagrams.

of contribution will consequently come out as a ultraviolet finite correction to the effective action. This in turn means that no such a term, which would for sure bring about longitudinal-mode ghosts, needs to be adjoined to the tree-level action. Therefore, since we know (based on power-counting) that from 3 loops on the model is totally finite, we can state that, though gauge-invariant, a term like $|(D_\mu B^\mu)|^2$ is not radiatively induced into the effective action. Therefore, the spectral conditions established in Section 2 are not spoiled whenever loop corrections are taken into account.

Appendix: 1-loop integrals for the CSM*-field self-energy

To solve the integrals I_1, I_2, I_3 and I_4 in Section 3, use has been made of the following well-known results [19] :

$$J_0 = \int \frac{d^D k}{(2\pi)^D} \frac{1}{(k^2 + 2p.k - c)^\alpha} = i(-1)^\alpha \frac{\pi^{\frac{D}{2}}}{(2\pi)^D}(c+p^2)^{\frac{D}{2}-\alpha} \times \times \frac{\Gamma(\alpha - \frac{D}{2})}{\Gamma(\alpha)} \ , \tag{27}$$

$$J_1^\mu = \int \frac{d^D k}{(2\pi)^D} \frac{k^\mu}{(k^2 + 2p.k - c)^\alpha} = i(-1)^{\alpha+1} \frac{\pi^{\frac{D}{2}}}{(2\pi)^D}(c+p^2)^{\frac{D}{2}-\alpha} \times \times p^\mu \frac{\Gamma(\alpha - \frac{D}{2})}{\Gamma(\alpha)} \tag{28}$$

and

$$J_2^{\mu\nu} = \int \frac{d^D k}{(2\pi)^D} \frac{k^\mu k^\nu}{(k^2 + 2p.k - c)^\alpha} = i(-1)^\alpha \frac{\pi^{\frac{D}{2}}}{(2\pi)^D}(c+p^2)^{\frac{D}{2}-\alpha} \times \times \left[\frac{\Gamma(\alpha - \frac{D}{2}) p^\mu p^\nu - \frac{1}{2}\Gamma\left(\alpha - 1 - \frac{D}{2}\right) \eta^{\mu\nu}(c+p^2)}{\Gamma(\alpha)} \right] . \tag{29}$$

We quote in the sequel the results for the Feynman parametric integrals performed after the integration over the loop momenta :

$$(I_1)_{\alpha\beta} = \begin{cases} i\frac{\omega^2}{8\pi M}\Big\{ p_\alpha p_\beta \left[-\frac{3(p^2+M^2)}{4(p^2)^2} M + \frac{3(p^2)^2+2M^2p^2+3M^4}{8(p^2)^2} W \right] + \\ + \eta_{\alpha\beta}\left[\frac{M^3}{4p^2} - \frac{(p^2-M^2)^2}{8p^2} W \right] \Big\} + i\frac{\omega^2}{32\pi}\eta_{\alpha\beta} \ \ , \ \ p^2 > 0 \\ i\frac{\omega^2}{8\pi M}\Big\{ p_\alpha p_\beta \left[-\frac{3(p^2+M^2)}{4(p^2)^2} M + \frac{3(p^2)^2+2M^2p^2+3M^4}{8(p^2)^2} V \right] + \\ + \eta_{\alpha\beta}\left[\frac{M^3}{4p^2} + \frac{(p^2-M^2)^2}{8p^2} V \right] \Big\} + i\frac{\omega^2}{32\pi}\eta_{\alpha\beta} \ \ , \ \ p^2 < 0 \end{cases} \tag{30}$$

$$(I_2)_{\alpha\beta} = \begin{cases} i\frac{\omega^2 M}{8\pi}\eta_{\alpha\beta} W \ \ , \ \ p^2 > 0 \\ i\frac{\omega^2}{8\pi M}\eta_{\alpha\beta}\left[\frac{3(p^2)^2-2M^2p^2+3M^4}{4p^2} V \right] \ \ , \ \ p^2 < 0 \end{cases} \tag{31}$$

$$(I_3)_{\alpha\beta} = \begin{cases} i\frac{\omega^2 M}{8\pi}\eta_{\alpha\beta}W \ , \ p^2>0 \\ i\frac{\omega^2 M}{8\pi}\eta_{\alpha\beta}V \ , \ p^2<0 \end{cases} \tag{32}$$

$$(I_4)_{\alpha\beta} = \begin{cases} i\frac{\omega^2}{8\pi M}\Big\{p_\alpha p_\beta\Big[-\frac{3(p^2+M^2)}{4(p^2)^2}M + i\frac{3\pi}{8}\frac{1}{\sqrt{p^2}} + \frac{3(p^2)^2+2M^2p^2+3M^4}{8(p^2)^2}W\Big] + \\ +\ \eta_{\alpha\beta}\Big[\frac{M^3}{4p^2} - i\frac{\pi}{8}\sqrt{p^2} - \frac{(p^2-M^2)^2}{8p^2}W\Big]\Big\} + i\frac{\omega^2}{32\pi}\eta_{\alpha\beta} \ , \ p^2>0 \\ i\frac{\omega^2}{8\pi M}\Big\{p_\alpha p_\beta\Big[-\frac{3(p^2+M^2)}{4(p^2)^2}M - \frac{3\pi}{8}\frac{1}{\sqrt{-p^2}} + \frac{3(p^2)^2+2M^2p^2+3M^4}{8(p^2)^2}V\Big] + \\ +\ \eta_{\alpha\beta}\Big[\frac{M^3}{4p^2} - \frac{\pi}{8}\sqrt{-p^2} + \frac{(p^2-M^2)^2}{8p^2}V\Big]\Big\} + i\frac{\omega^2}{32\pi}\eta_{\alpha\beta} \ , \ p^2<0 \end{cases} \tag{33}$$

where W and V are defined as

$$W \equiv \frac{1}{\sqrt{p^2}}\left[\ln(|p^2-M^2|) - 2\ln(|\sqrt{p^2}-M|) - i\pi\theta(p^2-M^2)\right] \quad \text{if } \ p^2>0 \tag{34}$$

and

$$V \equiv \frac{1}{\sqrt{-p^2}}\left[\frac{\pi}{2} - \arctan\left(\frac{p^2+M^2}{2M\sqrt{-p^2}}\right)\right] \quad \text{if } \ p^2<0\ . \tag{35}$$

Acknowledgements

The authors express their gratitude to Dr. J.A. Helayël-Neto, Dr. O. Piguet, Prof. M. Chaichian and Dr. S.P. Sorella for patient and helpful discussions. Thanks are also due to Prof. H. Freitas de Carvalho and our colleagues at CBPF-DCP. CNPq-Brazil and FAPERJ-Brazil are acknowledged for invaluable financial help.

References

[1] J. Schwinger, *Phys.Rev.* **125** (1962) 397 ;
J. Schwinger, *Phys.Rev.* **128** (1962) 2425.

[2] R. Jackiw and S. Templeton, *Phys.Rev.* **D23** (1981) 2291 ;
J. Schonfeld, *Nucl. Phys.* **B185** (1981) 157 ;
P.K. Townsend, K. Pilch and P. van Nieuwenhuizen *Phys.Lett.* **B136** (1984) 38 ;
S. Deser and R. Jackiw, *Phys.Lett.* **B139** (1984) 371.

[3] S. Deser, R. Jackiw and S. Templeton, *Phys.Rev.Lett.* **48** (1982) 975 ;
S. Deser, R. Jackiw and S. Templeton, *Ann.Phys. (N.Y.)* **140** (1982) 372.

[4] R.D. Pisarski, *Phys.Rev.* **D29** (1984) 2423.

[5] T.W. Appelquist, M. Bowick, D. Karabali and L.C.R. Wijewardhana, *Phys.Rev.* **D33** (1986) 3704 ;
T.W. Appelquist, D. Nash and L.C.R. Wijewardhana, *Phys.Rev.Lett.* **60** (1988) 2575.

[6] G.W. Semenoff and L.C.R. Wijewardhana, *Phys.Rev.Lett.* **64** (1989) 2633 ;
Y.-C. Kao, *Mod.Phys.Lett.* **A6** (1991) 3261.

[7] M. Koopmans and J. Steringa, *Phys.Lett.* **B226** (1989) 309.

[8] P.W. Higgs, *Phys.Lett.* **12** (1964) 132.

[9] S. Weinberg, *Phys.Rev.* **D7** (1973) 2887.

[10] Y. Nambu and G. Jona-Lasinio, *Phys.Rev.* **122** (1961) 345 ;
Y. Nambu and G. Jona-Lasinio, *Phys.Rev.* **124** (1961) 246.

[11] W.A. Bardeen, C.N. Leung and S.T. Love, *Nucl.Phys.* **B223** (1989) 493.

[12] S. Weinberg, *Understanding the Fundamental Constituents of Matter*, ed. A. Zichichi, Plenum Press (New York, 1978) ;
A. Linde, *Rep.Progr.Phys.* **42** (1979) 389 ;
D. Gross, R.D. Pisarski and L. Yaffe *Rev.Mod.Phys.* **53** (1981) 43.

[13] Y.H. Chen, F. Wilczek, E. Witten and B.I. Halperin, *Int.J.Mod.Phys.* **B3** (1989) 1001 ;
J.D. Lykken, *Chern-Simons and Anyonic Superconductivity*, talk given at the fourth annual Superstring Workshop, "Strings 90", Texas A & M University (Texas, March 1990).

[14] F. Delduc, C. Lucchesi, O. Piguet and S.P. Sorella, *Nucl.Phys.* **B346** (1990) 313 ;
A. Blasi, O. Piguet and S.P. Sorella, *Nucl.Phys.* **B356** (1991) 154 ;
C. Lucchesi and O. Piguet, *Nucl.Phys.* **B381** (1992) 281.

[15] J. Wess and J. Bagger, *Supersymmetry and Supergravity*, Princeton Univ. Press (Princeton, 1983).

[16] O.M. Del Cima, *Mod.Phys.Lett.* **A9** (1994) 1695 ;
J.A. Helayël-Neto, O. Piguet, M. Chaichian and S.P. Sorella, *private communications.*

[17] O.M. Del Cima, **M.Sc. Thesis**, *Aspects of the Dynamics of Complex Vector Fields in 3 Dimensions*, CBPF (July, 1993).

[18] O. Piguet and J.A. Helayël-Neto, *private communications.*

[19] J.C. Collins, *Renormalization*, Cambridge Univ. Press (Cambridge, 1984).